# Logical Characterization of Hereditary History-preserving Bisimulation over Higher Dimensional Automata*

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**Abstract.** This talk presents a work in progress on the relation between a modal logic with past modalities and hereditary history-preserving bisimulation in the setting of higher dimensional automata. This modal characterization result comes to strengthen a preliminary relation between higher dimensional modal logic and split-bisimulation, over the same concurrency model. The motivation for this study is to eventually develop modal characterizations for all concurrency bisimulations in between split- and hh-bisimulation, as syntactic restrictions of our modal logic.

**Keywords:** Bisimulation · Higher dimensional automata · higher dimensional modal logic · hereditary history-preserving bisimulation.

## 1 Higher dimesional automata and bisimulation

Introduced by Pratt and van Glabbeek [6,15], Higher Dimensional Automata (HDA) are a highly expressive model of non-interleaving concurrency, which has many concurrency models as instances, e.g., event structures [18], asynchronous transition systems, or Petri nets. Applications of HDA ranged over scheduling problems, wait-free protocols for distributed systems [10] [11] [9], and model-checking [4]. In [12] , it is shown that HDA are promising for solving state-explosion problem

HDA are geometrical in nature, with varied representations influenced by the algebraic topology concept of cubical sets.

**Definition of Higher Dimensional Automata** [6]

A cubical set consists of a family of sets $(\mathbf{Q}_n)_{n\geqslant 0}$ and for every $n \in \mathbf{N}$ a family of maps $s_i, t_i \;:\; \mathbf{Q}_n \to \; \mathbf{Q}_{n-1}$ for every $1 \leqslant i \leqslant n$, such that

$$\alpha_i \circ \beta_j = \beta_{j-1} \circ \alpha_i \; for \; all \; 1 \leqslant i < j \leqslant n \; and \; \alpha, \beta \in \{s, t\} \tag{1}$$

A HDA, labelled over an alphabet A, is a tuple $(\mathbf{Q}, s, t, I, F, \lambda)$ with $\mathbf{Q} = \bigcup_{k=0}^{\infty} \mathbf{Q}_k$, $(\mathbf{Q}, s, t)$ a cubical set, $I \in \mathbf{Q}_0$, $F \subseteq \mathbf{Q}_0$ ,and $s$ and $t$ denote the families of all maps $s_i$ and $t_i$.

Intuitively, a Higher Dimensional Automaton is a collection of hypercubes (in all dimensions) glued together in possibly convoluted and cyclical structures. In terms of concurrency, the 3-dimensional cube represents three events executing concurrently, whereas the hollow cube (i.e., only the six 2-dimensional square faces) models the 'two-by-two' concurrency of the same three events.

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When restricting HDA to only their 1-dimensional objects we obtain the well known automata (or labelled transition systems).

Transition systems come with bisimulations as their notion of behavioural equivalence (different notions of equivalence depending on what behaviours one is interested in observing). Likewise, HDA also have various concurrency bisimulations. These are bisimulations that have originally been introduced for other non-interleaving models and then adopted for HDA, such as history preserving bisimulation, hereditary history preserving bisimulation and ST-bisimulation (see [6] for an overview).

## 2 Higher dimensional modal logic and hh-bisimulation

Standard modal logic [3] is a powerful and flexible tool for reasoning about relational structures such as labeled transition systems. Many variants of modal logic exist, including temporal logics [1] or dynamic logics [13]. A modal logic for HDA has been introduced in [16] which when suitably restricted becomes the standard modal logic over the 1-dimensional restriction of HDA. This higher dimensional modal logic (HDML) has two types of forward modalities, matching the types of execution steps existing inside an HDA.

The language of HDML is constucted from a set of atomic formulas $\Psi$ and given by :

$$\phi := \psi \mid \bot \mid \phi \to \phi \mid \langle|\varphi \mid |\rangle\varphi$$

Where $\psi \in \Psi$ , $a \in A$ , $\langle a|$ the during modality and $|a\rangle$ the after modality. The models of HDML is a higher dimensional structure $\mathcal{H}$, such as HDA, together with a valuation $\mathcal{V}$ that associates a set of atomic propositions to each n-cells.

The satisfaction relation of a HDML formula is defined recursively in n-cells as follows:

- $\mathcal{H}, q \models \phi$ iff $\phi \in \mathcal{V}(q)$
- $\mathcal{H}, q \not\models \bot$
- $\mathcal{H}, q \models \phi \to \psi$ iff $\mathcal{H}, q \models \psi$ whenever $\mathcal{H}, q \models \phi$
- $\mathcal{H}, q \models \langle|\varphi, q \in Q_n$ iff $\exists q' \in Q_{n+1},\ s_i(q') = q, 1 \le i \le n+1$ *and* $q' \models \phi$
- $\mathcal{H}, q \models |\rangle\varphi, q \in Q_n$ iff $\exists q' \in Q_{n-1},\ t_i(q) = q', 1 \le i \le n$ *and* $q' \models \phi$

One standard result for modal logics is to study their expressiveness by identifying the precise bisimulation that it captures, i.e., two states of a transition system are bisimilar precisely when they are modally equivalent, by which it is meant that they satisfy the same formulas. Such modal characterizations are well known for the bisimulations of the sequential world, e.g., see the spectrum of van Glabbeek [5] where each bisimulation has a particular modal logic associated.

In this talk, we introduce a preliminary result that provides a logical characterization between HDML and split-bisimulation, which is quite low in terms of expressiveness among concurrency bisimulations (see e.g., [8]). Furthermore, we will present a work in progress that characterizes hereditary history-preserving bisimulation, which is one of the more well behaved and fine-grained equivalence over HDA (see also [7]), based on an extension of HDML with past modalities introduced in [17], namely h-HDML.

The main idea is to use past modalities (well known from temporal logics) to be able to match the expressiveness of hh-bisimulation, which has access to the whole partially ordered history of each execution path in an HDA. We will present the main elements of the proof as well as examples of formulas of increasing complexity that are used to distinguish various examples taken from the literature on concurrency bisimulations; all of these in the setting of HDA and of the extended HDML.

The ultimate goal of this work is to provide modal characterizations of all concurrency bisimulations in between split- and hh-bisimulation, similar in nature to the modal logics given by van Glabbeek for his spectrum [5], where HDML would sit quite low in terms of expressiveness and its extension with past modalities (called hHDML) quite high. In between we would expect to find (syntactical) restrictions of hHDML or eventually a restriction of one of its variants. This approach could be seen as an opposite to how Van Glabbeek works, where instead we start from a highly expressive modal logic and identify the precise restrictions in terms of logical constructs needed to drop some of the expressiveness of the hh-bisimulation. Another benefit of this work is related to the role of HDA as a model that includes other concurrency models as classes of restricted HDA. Our logical characterizations should then become equivalent to existing modal logics over event structures [2] or configuration structures [14] and their respective concurrency bisimulations.

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